Tax Increment Financing: A Theoretical Inquiry

by

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Abstract

This paper offers an analysis of tax increment financing, adding to a small theoretical literature on this important fiscal instrument. The analysis exploits the theoretical connection between property values and public-good levels, which is the subject of a large literature in local public finance. Using this approach, the paper shows that localized public improvements are likely to be opposed by property owners outside the affected area, who pay higher property taxes with no offsetting benefits. By using tax revenue captured from overlapping jurisdictions, TIF may circumvent this opposition, allowing the city to implement the public improvement without an increase in its tax rate. TIF is not always viable as a financing method, however, because it may not generate enough additional revenue. The analysis shows that TIF's viability is ensured only when the public good is at least moderately underprovided relative to the socially-optimal level. In the case where the public good is slightly underprovided, a public improvement is desirable, but TIF is not viable. Finally, the analysis shows that the public-good levels ultimately chosen under TIF need not be efficient, with both under- and overprovision being possible outcomes. Thus, while TIF may allow a city to carry out needed public improvements, the stimulus it provides may be excessive.

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1. Introduction

Tax increment financing (TIF) is now widely used by local governments in the U.S. TIF allows a city government or other local jurisdiction to finance an increase in the level of a public good in a given neighborhood without raising the property-tax rate it charges. No tax increase is necessary because, under TIF, the city captures a portion of the tax revenue accruing to other overlapping jurisdictions. This captured revenue, which comes from the neighborhood where the public improvement takes place, is used to help defray its cost.

To see in more detail how TIF works, observe that the public improvement, which usually upgrades the level of infrastructure, raises property values in the affected neighborhood (the "TIF zone"). While these higher values generate an increase in the city's property-tax revenue, the extra funds are typically insufficient to cover the cost of the improvement. However, higher property values in the TIF zone also generate more tax revenue for overlapping jurisdictions such as the school district, park district, etc. TIF diverts this additional revenue to the city, keeping it out of the coffers of the other local governments. Combined with the additional revenue from the city's own property tax, the captured revenue helps pay for the improvement.

As noted by Chapman (1998), 46 states have adopted statutes allowing the use of TIF by local governments. In these states, nearly all cities with populations above 50,000 contain one or more TIF zones, which exhibit a wide variety of public improvements. While TIF is extremely popular with local government officials, its widespread use is not without controversy. For example, although the typical enabling statute requires that a TIF zone constitute a "blighted" area, the relative economic health of some zones has raised allegations that TIF authority is misused. Officials of jurisdictions whose tax revenue is captured by TIF are likely to raise such allegations, claiming that TIF provides a way for the city (or other implementing jurisdiction) to "steal" their funds. Controversy also centers around the question of whether TIF-financed

improvements actually raise property values, as claimed by proponents. A related issue is whether TIF is indeed self-financing, as billed, or whether taxpayers outside the TIF zone must ultimately help defray the cost of the improvement with higher property-tax payments.

These institutional issues are clearly discussed by Chapman (1998), and many of the same points are raised in the earlier paper of Huddleston (1984). In addition to these descriptive studies, a small empirical literature has emerged, focusing on the effects of TIF and on the forces leading to its adoption. Anderson (1990) shows that TIF tends to be adopted in cities that have relatively high growth rates of property values, which suggests that the "revenue stealing" motivation mentioned above may sometimes guide TIF adoption. Man and Rosentraub (1998) and Dye and Merriman (1999) explore the effect of TIF on property-value growth, recognizing that the adoption decision itself is endogenous. These studies reach contradictory conclusions, with the first showing the predicted positive effect of TIF on the growth rate of values and the second showing a negative effect. Dye and Merriman (1999) ascribe this negative effect to a TIF-induced distortion in the location of real estate investment, which may be channeled by TIF to relatively unproductive locations within the city. Like Man and Rosentraub (1998), Wassmer (1994) finds a positive effect from local development incentives, including TIF, although he does not control for the endogeneity of these incentives.

Despite the conceptual complexity of TIF, little effort has been devoted to theoretical analysis of this important fiscal instrument. The only theoretical studies are by Dye and Sundberg (1998) and Donaghy, Elson and Knaap (1999). While both studies propose and analyze sensible models of TIF, neither incorporates the theoretical link between property values and public-good provision, which is analyzed in the large literature on local public finance. This link is the engine that drives TIF, and a model that incorporates it may give useful insights beyond those provided by these earlier papers.

The purpose of the present paper is to develop such a model of TIF, drawing on the earlier work of Brueckner (1979, 1982, 1983). The analysis is directed toward answering several questions. The first question concerns the political impetus for the adoption of TIF. This impetus can be clarified by identifying the gainers and losers when a marginal public improvement is carried out *without* the use of TIF. The analysis shows that, in the absence of TIF, the owners

of property in the affected zone gain, enjoying higher values. However, owners outside the zone, whose properties do not benefit from the improvement, suffer a loss in value. This loss arises because, in the absence of TIF, the city typically must raise its property-tax rate to finance the improvement, which depresses property values outside the zone. While these losses may be offset by gains within the zone, so that *aggregate* property values in the city rise, political opposition by the losers may block the improvement, preventing these social gains from being realized. As a result, some alternate method for financing the improvement that does not raise taxes for the opposing property owners must be found. Since TIF may offer such a method, the political impetus for its adoption is clear.

This leads to the next question, namely whether TIF is indeed viable in the sense of being self-financing. In other words, the question is whether use of TIF generates enough tax revenue to cover the city's cost of the marginal public improvement. Letting z denote the level of the city-provided public good, this question can be answered by referring to an important benchmark: the socially-optimal level of z, denoted z^*. At this level of provision, the marginal social benefit from an increase in z equals the marginal cost.

The analysis shows that if the public good in the TIF zone is initially overprovided, with z above z^*, then self-financing fails. In this case, TIF does not generate enough revenue to cover the cost of increasing the public good above its initial level. The same conclusion holds if the good is slightly underprovided, with z initially below z^* but relatively close to it. Again, a marginal increase in the public-good level is not self-financing. The analysis shows that TIF is viable, with a marginal public improvement guaranteed to be self-financing, only if the public good is moderately or seriously underprovided, with the initial level of z *well below the socially-optimal level.*

This conclusion is encouraging because it shows that TIF will work only in those parts of a city where public goods are noticeably underprovided. But those areas are the natural targets of TIF, as envisioned in the typical enabling statute. Conversely, the analysis shows that TIF fails in zones where the need for a public improvement is nonexistent or weak. These are zones where the public good is overprovided or only slightly underprovided.

The analysis also explores how the critical value of z, above which self-financing fails,

is affected by the fiscal environment. The critical value is shown to depend on the number of overlapping jurisdictions whose tax revenue is captured under TIF, and on the average property-tax rate among them.

The last question concerns the optimality of the public-good level ultimately chosen in the TIF zone. Assuming that z is chosen to benefit the zone's property owners, the analysis shows that its level will be pushed to the point where a further increase creates a budget deficit. The resulting z level, however, could be either higher or lower than z^*, the zone's socially-optimal level. Thus, TIF need not lead to efficient provision of the public good.

It should be noted that these results are generated using a static model. Thus, unlike in the other theoretical studies cited above, property values are constant over time in the model, changing only in response to the public improvement. It can be shown that relaxing this assumption has no effect on the analysis, provided that values grow with an exponential time trend.

The plan of the paper is as follows. Section 2 presents the model and explores the political forces leading to the emergence of TIF. Section 3 analyses the self-financing issue and considers the optimality of the public-good level ultimately chosen under TIF. Section 4 offers conclusions.

2. The Model and the Impetus for TIF

2.1. Model

To develop the model, suppose that the city is divided into two neighborhoods, a and b, and that it provides a single public good z. Suppose, however, that the public good is excludable, so that it may be provided at different levels in the two neighborhoods, with the levels denoted z_a and z_b. Spillovers across the neighborhoods are assumed to be absent, so that public consumption in neighborhood a is independent of z_b and vice versa. While a no-spillover assumption may be accurate for some types of public goods (schools, neighborhood parks), it may be violated by others (police and fire protection, roads), where the quality of service in one neighborhood may affect residents in adjoining areas. The assumption can be relaxed at the cost of additional complexity in the analysis.

The total cost of providing public goods z_a and z_b to the neighborhoods is given by $C_a(z_a) + C_b(z_b)$, where the individual cost functions $C_a(\cdot)$ and $C_b(\cdot)$ are convex. Differences between these functions reflect dissimilarities in the sizes and other characteristics of the two neighborhoods. It should be noted that separability of costs across neighborhoods is assumed only for expository purposes. All of the results continue to hold if costs are nonseparable.

Residents of the city also consume another public good s that is provided by a separate jurisdiction, which can be thought of as the school district. The city is assumed to be coterminous with the school district, although the analysis only requires that the eventual TIF zone, which corresponds to neighborhood a, lies within the district. For simplicity, the level of s is assumed to be the same across neighborhoods a and b. The cost of providing s is given by $K(s)$, a convex function.

The model is developed under the assumption that property in the city is entirely residential. Once the analysis is complete, however, it is shown that all of the results generalize to the more realistic case where the city has a mixture of both residential and nonresidential property. Residents of the city are assumed to have identical incomes and preferences, given by a common utility function $U(\cdot)$, and all are assumed (without loss of generality) to rent the houses they occupy.

Let the houses in the city be indexed by i, and let q_i denote the vector of attributes of house i. Then the utility of house i's occupant is given by $U(x_i, q_i, z, s)$, where x_i is the individual's consumption of a numeraire private good, and where $z = z_a$ if house i is in neighborhood a, with $z = z_b$ otherwise. Letting y denote the common income level of city residents and P_i denote the rent payment for house i, the budget constraint of its occupant is given by $x_i + P_i = y$.

Combining the above elements, the rent payment P_i can be determined using a "bid-rent" approach, following Wheaton (1977) and Brueckner (1979, 1982). This approach assumes that residents of the city, who are freely mobile, enjoy a fixed utility level equal to that available elsewhere in the economy. Letting $\overline{U}$ denote this utility level, and eliminating x_i via the budget constraint, the fixed-utility requirement for a resident of neighborhood a may be written $U(y - P_i, q_i, z_a, s) = \overline{U}$. This condition implicitly defines P_i as a function of z_a, q_i,

and s, with derivatives $\partial P_i/\partial z_a = U_i^z/U_i^x$, $\partial P_i/\partial q_i = U_i^q/U_i^x$, and $\partial P_i/\partial s = U_i^s/U_i^x$ (the U superscripts denote partial derivatives; the i subscripts capture variation across households in U's arguments). Thus, as housing attributes or the public-good levels increase, the rent for house i increases at a rate equal to the marginal rate of substitution (MRS) between the relevant good and x. Analogous results, with z_b in place of z_a, apply if house i is located in neighborhood b.

As in the case of costs, a separability assumption is imposed on preferences in order to simplify the discussion. In particular, the utility function is assumed to be additively separable in its z, s, and x arguments. This means that the MRS between z and x is independent of s, and that the MRS between s and x is independent of z. As will be seen below, this property means that the socially-optimal levels of z_a and s can be determined independently of one another, with the same point applying to z_b and s.

The analysis ultimately explores the effect on property values of a marginal public improvement in neighborhood a. With such an improvement, the city-provided good z_a increases marginally while the good s provided by school district is held fixed. In addition, the levels of housing attributes q_i for all the houses in neighborhood a are held fixed as z_a increases. The latter assumption, which rules out real estate investment in response to the public improvement, is imposed to facilitate the exposition. However, since upgrading of existing property and new construction are major goals of TIF-financed projects, these outcomes must be incorporated in any realistic model. This is done below once the main results of the analysis are established. It is shown that models with and without new investment are formally equivalent, so that all the results continue to apply when housing attributes change in response to the public improvement.

With s and the housing attributes fixed, rent can be written solely as a function of z, with the rent level for house i written as $P_i(z_a)$ if the house is located in neighborhood a and $P_i(z_b)$ it is located in b. Total rent in each of the neighborhoods is then given by

$$R_a(z_a) \equiv \sum_{i \in a} P_i(z_a) \; ; \qquad R_b(z_b) \equiv \sum_{i \in b} P_i(z_b). \tag{1}$$

Using the above results, the derivatives of R_a and R_b are given by

$$R_a'(z_a) = \sum_{i \in a} \mathrm{MRS}_i^{z,x} \; ; \qquad R_b'(z_b) = \sum_{i \in b} \mathrm{MRS}_i^{z,x}, \tag{2}$$

where $\mathrm{MRS}_i^{z,x} = U_i^z / U_i^x$. Therefore, total rent in a neighborhood increases as its z level rises, with the increase equal to the sum across houses of the marginal benefit from the extra z. The neighborhood rent gain thus reflects the marginal social benefit of the extra public good. In addition, it can also be shown that when the utility function has strictly convex indifference surfaces, the second derivatives R_a'' and R_b'' are both negative, indicating strict concavity of the rent functions. Note also that, while the levels of R_a and R_b implicitly depend on s, the previous separability assumption on preferences means that the derivatives R_a' and R_b' are independent of s.

The value of houses in the city depends on the rents they generate and on the property taxes they pay. The city levies property taxes at rate τ, which is common across the neighborhoods, while the school district's tax rate is given by t. Letting V_a and V_b denote the total values of houses in the two neighborhoods, these values are defined by the relationships $V_a = [R_a - (\tau + t)V_a]/r$ and $V_b = [R_b - (\tau + t)V_b]/r$, where r is the discount rate. Note that these equations indicate that property value equals the present discounted value of rental income minus property taxes. Solving for V_a and V_b yields

$$V_a = \frac{R_a(z_a)}{\tau + \theta} \tag{3}$$

$$V_b = \frac{R_b(z_b)}{\tau + \theta}, \tag{4}$$

where $\theta = t + r$. Neighborhood value thus equals total rent divided by the sum of the two property-tax rates and the discount rate.

Without loss of generality, both the city and school district are assumed to rely entirely on property taxes to finance their expenditures. As a result, the city's budget constraint can be written $\tau(V_a + V_b) = C_a(z_a) + C_b(z_b)$. Note that since the city realistically pools the tax revenue

from both neighborhoods, a given neighborhood's revenue need not cover its public-good costs even though the overall budget must balance. Using (3) and (4), this budget constraint can be rewritten as

$$\frac{\tau[R_a(z_a) \ + \ R_b(z_b)]}{\tau+\theta} \quad = \quad C_a(z_a) \ + \ C_b(z_b). \tag{5}$$

Similarly, the school district's budget constraint is given by $t(V_a + V_b) \ = \ K(s)$, which leads to an equation analogous to (5).

Equation (5) can be rearranged to solve for the city's budget-balancing tax rate, which equals $\tau = \theta(C_a + C_b)/(R_a - C_a + R_b - C_b)$. The effect of an increase in z_a on τ is important in the ensuing discussion, and this effect is found by differentiating the previous equation. The result is

$$\frac{\partial\tau}{\partial z_a} \quad = \quad \frac{\theta(R_a \ + \ R_b)\left(C_a' \ - \ \frac{\tau}{\tau+\theta}R_a'\right)}{(R_a \ - \ C_a \ + \ R_b \ - \ C_b)^2}. \tag{6}$$

The derivative $\partial\tau/\partial z_b$ is given by the analogous expression with C_b' and R_b' in place of C_a' and R_a'. The sign of (6) is discussed below.

Since the utilities of its residents are fixed, the proper goal for the city is to choose its public-good levels in the interest of property owners. In an ideal world, the city would then set these levels so as to maximize the total value of property across both neighborhoods. The city would thus choose z_a and z_b to maximize $V_a + V_b = (R_a + R_b)/(\tau + \theta)$, using (6) and the analogous expression for $\partial\tau/\partial z_b$ to account for the required budget-balancing changes in τ. The first-order condition for z_a is

$$\frac{d(V_a + V_b)}{dz_a} \quad = \quad \frac{R_a'}{\tau+\theta} \ - \ \frac{R_a + R_b}{(\tau+\theta)^2}\frac{\partial\tau}{\partial z_a} \quad = \quad 0. \tag{7}$$

After substituting (6), this condition reduces to

$$R_a'(z_a) \quad = \quad C_a'(z_a), \tag{8}$$

which says that the marginal change in rent from an increase in z_a should equal the cost of the extra public good.[1] But recalling (2), condition (8) reduces to the Samuelson optimality

condition $\sum_{i \in a} \mathrm{MRS}_i^{z,x} = C_a'(z_a)$, which says that marginal social benefit from an increase in the public good equals marginal cost. An analogous condition emerges in neighborhood b, and a city-wide Samuelson condition is generated if s and t are also chosen to maximize aggregate property value. Therefore, property-value maximization leads to efficient provision of public goods, as originally demonstrated by Brueckner (1982, 1983) and other authors.

For future reference, the socially-optimal public-good level in neighborhood a is denoted z_a^*, and it satisfies $R_a'(z_a^*) = C_a'(z_a^*)$. The public good is said to be *underprovided* when $z_a < z_a^*$ and *overprovided* when $z_a > z_a^*$. Since $R_a'' < 0$ and $C_a'' \geq 0$ hold, underprovision of z_a implies $R_a' > C_a'$, while overprovision implies $R_a' < C_a'$. It is important to note that, because of the separability of preferences, the z_a^* that solves $R_a'(z_a^*) = C_a'(z_a^*)$ does not depend on the level of s, indicating that the socially-optimal z_a can be determined without reference to s. This differs from the general case, where the optimal levels of multiple public goods must be determined simultaneously.

Although property-value maximization generates an ideal outcome, the public-good levels in actual communities may not always be chosen according to this principle (see Brueckner (1979, 1982) for evidence). Accordingly, the ensuing discussion focuses on the effect of an increase in z_a assuming that its initial level, along with the levels of z_b and s, are *set arbitrarily.* From the above discussion, the social desirability of a change in z_a will not depend on the magnitude of the fixed s, a key simplification that follows from the separability of preferences.

2.2. The impetus for TIF

The efficiency condition in (8) is derived by maximizing aggregate property value in the city, which is appropriate when property owners are viewed as a single interest group. However, to investigate the impetus for TIF, it is useful to treat the owners of property in the two neighborhoods as *separate* interest groups. The effects of a marginal increase in z_a can then be appraised separately for the two groups, generating some important insights. In carrying out this exercise, a key assumption is that the decision variables of the school district, namely its public-good level s and tax rate t, are *held fixed.* The implications of this assumption are discussed below.

The impacts of a higher z_a on the two groups are distinguished by a key difference between

the neighborhoods. In particular, while rents in neighborhood a rise as a result of the public improvement, rents in neighborhood b are unaffected since z_b remains constant. The groups, however, are symmetrically affected by the change in the city's property-tax rate. Suppose that τ must rise along with z_a to maintain the city's budget balance, as intuition might suggest. Then, since property values are decreasing in τ while z_b is fixed, (4) shows that *values fall in neighborhood b* as a result of the public improvement in neighborhood a. Since neighborhood-b owners are thus made worse off by the improvement, they may take political action to prevent its implementation. The existence of this opposition suggests that, if the improvement is to take place, another financing method that does not require an increase in τ may be needed.

While this conclusion identifies a possible impetus for TIF, several questions remain unaddressed. First, will the public improvement indeed require an increase in τ, as assumed? Second, will neighborhood a's property owners benefit from the improvement, so that at least one group favors it? The first question can be answered by referring to (6), which shows that

$$\frac{\partial \tau}{\partial z_a} \;>\; (<)\; 0 \qquad \text{as} \qquad \frac{\tau}{\tau+\theta} R_a' \;-\; C_a' \;<\; (>)\; 0. \tag{9}$$

To understand the implications of (9), observe first the expression $\tau R_a'/(\tau+\theta) - C_a'$ can change sign at most once (from positive to negative) as z_a increases. The proof of this fact takes into account the effect of z_a on τ, as well as its effect on R_a' and C_a'.[2] Given this result, it follows that the above expression is either negative for all $z_a > 0$, or that it changes sign from positive to negative at some "τ-critical value" $\widehat{z}_a^{\tau} > 0$, so that

$$\frac{\tau}{\tau+\theta} R_a' \;-\; C_a' \;<\; (>)\; 0 \qquad \text{as} \qquad z_a \;>\; (<)\; \widehat{z}_a^{\tau}. \tag{10}$$

For (10) to hold, it must be the case that the left-hand side evaluated at $z_a = 0$ is positive. In other words,

$$\frac{\tau_0}{\tau_0+\theta} R_a'(0) \;-\; C_a'(0) \;>\; 0 \qquad \text{(Condition } \Gamma\text{)} \tag{11}$$

must hold, where τ_0 gives the city's tax rate when $z_a = 0$.[3] The inequality in (11), denoted condition Γ, will be satisfied when the marginal social benefit of the first unit of z_a, as reflected

by $R_a'(0)$, is sufficiently large relative to its marginal cost, $C_a'(0)$. If the public good contributes importantly to consumer well being, condition Γ is likely to hold. Its satisfaction is assumed in the ensuing discussion.

When condition Γ holds, combining (10) with (9) yields

$$\frac{\partial \tau}{\partial z_a} \quad > \ (<) \ \ 0 \qquad \text{as} \qquad z_a \quad > \ (<) \ \ \widehat{z}_a^{\tau}. \tag{12}$$

Thus, the public improvement requires an increase (decrease) in the city's tax rate if the initial level of z_a is above (below) the τ-critical value $\widehat{z}_a^{\tau}$.

It is easily seen that the τ-critical value lies below z_a^*, the socially optimal level of z_a, so that

$$\widehat{z}_a^{\tau} \quad < \quad z_a^*. \tag{13}$$

This inequality is established by noting that $\widehat{z}_a^{\tau}$ satisfies the equation $\alpha R_a'(z_a) - C_a'(z_a) = 0$, where $\alpha = \tau/(\tau + \theta) < 1$, while z_a^* satisfies the same equation with α replaced by 1. Given $R_a'' < 0$ and $C_a'' \geq 0$, these facts can be used to establish (13).[4]

Combining (12) and (13), $\partial\tau/\partial z_a$ is then positive, indicating that the public improvement requires an increase in τ, when z_a is above a critical value that lies below the socially-optimal level. Since $\partial\tau/\partial z_a < 0$ holds only if $z_a < \widehat{z}_a^{\tau} < z_a^*$, it follows that the tax rate can be reduced as z_a rises only if the public good is *seriously underprovided*, with the initial z_a well below the socially-optimal level. Otherwise, τ rises with z_a.

To see the intuitive explanation for these results, note that when τ is held constant, tax revenue increases as z_a rises. The increase in revenue, which is due to the higher property values caused by higher rents, is given by $\tau R_a'/(\tau + \theta)$, as can be seen from (3). However, if z_a is seriously underprovided, then R_a' is much larger than marginal cost C_a'. In this case, the revenue increase $\tau R_a'/(\tau + \theta)$ may exceed marginal cost even though the multiplicative term $\tau/(\tau + \theta)$ is less than one, an outcome that is ensured when condition Γ holds. When this is true, the increase in tax revenue more than covers the cost of a marginal increase in z_a, *holding τ constant*. To avoid the resulting budget surplus, τ must then be *decreased* as

z_a rises, as seen in second inequality of (12). If, on the other hand, z_a is only moderately underprovided or overprovided, then $\tau R_a'/(\tau+\theta)$ is less than C_a', indicating that the revenue increase from the improvement falls short of marginal cost, holding τ constant. In this case, the public improvement requires an *increase* in the city's tax rate.

Since a higher τ is needed unless z_a is seriously underprovided, this discussion shows that the public improvement in neighborhood a will typically hurt property owners in neighborhood b, as mentioned above. It remains to gauge the effect of the improvement on neighborhood a's owners. Using (3), the effect of a marginal increase in z_a on property values in neighborhood a is given by

$$\frac{dV_a}{dz_a} = \frac{R_a'}{\tau+\theta} - \frac{R_a}{(\tau+\theta)^2}\frac{\partial\tau}{\partial z_a}. \tag{14}$$

After substituting (6), (14) reduces to an expression with the sign of

$$(R_a' - C_a')R_a + \frac{\theta}{\tau+\theta}R_a'R_b. \tag{15}$$

The expression in (15) is positive when $z_a < z_a^*$, since this ensures $R_a' - C_a' > 0$. Thus, if z_a is initially underprovided, a marginal increase raises neighborhood a's property values. Property values also rise if z_a is slightly overprovided, in which case the first term in (15) is negative but dominated by the positive second term. However, because (15) need not be monotonically decreasing in z_a, it is not possible to establish the existence of a critical value lying above z_a^*, which separates ranges where (15) is positive and negative. The most that can be said is that neighborhood a's property values may or may not rise with the public improvement when z_a is initially above z_a^*.

Summarizing the above discussion yields[5]

Proposition 1. *Unless the public good in a neighborhood is seriously underprovided, a marginal public improvement in the absence of TIF requires an increase in the city's tax rate, hurting property owners in other neighborhoods. But property owners in the affected neighborhood typically gain, an outcome that is assured if the public good is initially underprovided. Therefore, while property owners who are directly affected will typically support a public improvement, those in other neighborhoods will tend to oppose it.*

Since the city's aggregate property value is maximized when $z_a = z_a^*$, it follows that a marginal public improvement raises aggregate value when z_a is initially underprovided. The above discussion shows that, despite these generalized gains, localized losses are likely to emerge in neighborhood b.[6] Because of these losses, the affected property owners may block the improvement, preventing its social benefits from being realized. Their opposition would be defused, however, if the public improvement could be financed in a way that avoids the need for an increase in the city's tax rate. As seen in the next section, TIF may provide such a financing method.

Before turning to the analysis of TIF, an issue that has been omitted in the previous discussion must be considered, namely the effect of the public improvement on the fiscal position of the school district. If z_a is initially underprovided, then as noted above, the public improvement raises aggregate property value in the city. But a higher aggregate value in turn leads to higher tax revenue for the school district, creating a budget surplus. The surplus might be eliminated by reducing t, the school tax rate, an action that would benefit property owners in both neighborhoods. The resulting property value gains would generate a further beneficial feedback by allowing a slight reduction in τ by the city. The upshot is that, once school-district adjustments are taken into account, the τ-critical value $\widehat{z}_a^\tau$ may be too pessimistic in demarcating the areas of gains and losses for neighborhood-b property owners. In other words, when t is adjusted along with τ, neighborhood-b owners may gain from the public improvement for a range of z_a values that extends above $\widehat{z}_a^\tau$.

Despite this complication, the critical value $\widehat{z}_a^\tau$ may still be the appropriate reference point when the city evaluates the political feasibility of a public improvement. The reason is that, by adopting a partial equilibrium view and ignoring the school-district budget surplus that may be created by its action, the city avoids the need to speculate about what steps the school district would take to eliminate the surplus (these could involve a cut in t or an increase in s, or some combination of the two). Thus, in appraising the political support for its public improvement, the city plausibly views the decision variables of the school district as fixed, making $\widehat{z}_a^\tau$ the relevant critical value.

3. Analysis of TIF

3.1. Viability of TIF

To defuse anticipated opposition from neighborhood-b property owners, the city must find a way of carrying out the public improvement without raising its tax rate. But from above, the city cannot keep τ fixed at its initial level, relying solely on the incremental tax revenue generated from the higher rents caused by the improvement. This revenue is equal to $\tau R_a'/(\tau+\theta)$, which falls short of the improvement's cost when $z_a > \widehat{z}_a^{\tau}$. In other words, recalling $\theta = t+r$ and rewriting (10),

$$\text{incremental city tax revenue} \quad = \quad \frac{\tau}{\tau+t+r}R_a' \quad < \quad C_a' \quad \text{when} \quad z_a \quad > \quad \widehat{z}_a^{\tau}. \tag{16}$$

The solution is for the city to apply for TIF authority, with neighborhood a designated as the TIF zone. Then, the incremental tax revenue accruing to other jurisdictions, which arises from higher rents in the TIF zone, can be captured by the city and used to defray the cost of the improvement. In the present context, the relevant incremental revenue is earned by the school district and is given by $tR_a'/(\tau+\theta)$, which equals the fixed school tax rate times the rent-driven increase in property values in the TIF zone. Note that increasing z_a while holding τ constant generates a budget surplus for the school district similar to that discussed above but larger in size.[7] While the city ignored the previous surplus, the surplus funds generated under TIF are captured by the city.

Adding the captured school-district revenue to the city's own revenue, the public improvement under TIF generates a tax-revenue increase equal to

$$\frac{\tau}{\tau+\theta}R_a' \;+\; \frac{t}{\tau+\theta}R_a' \quad = \quad \frac{\tau+t}{\tau+t+r}R_a'. \tag{17}$$

For TIF to be viable as a means of financing the improvement, the revenue gain in (17) must exceed the cost of increasing z_a. Viability, which means that TIF is self-financing, thus requires

$$\frac{\tau+t}{\tau+t+r}R_a' \quad > \quad C_a'. \tag{18}$$

Since the factor multiplying R_a' is larger in (18) than in (16), it is evident that (18) may hold, indicating TIF's viability, at the same time that (16) is satisfied, indicating that the city alone, holding its tax rate fixed, cannot cover the cost of the improvement. To evaluate the conditions under which (18) is satisfied, the first step is to note that $(\tau+t)R_a'/(\tau+t+r)-C_a'$ changes sign from positive to negative as z_a increases.[8] Because the presence of t in the numerator makes this expression larger than the corresponding expression in (10), it follows that the expression changes sign at a critical value $\widehat{z}_a^{TIF}$ that is *larger* than the previous critical value $\widehat{z}_a^{\tau}$. Thus, (18) holds, indicating that TIF is viable, when z_a is less than the "TIF-critical value" $\widehat{z}_a^{TIF}$, which itself lies above $\widehat{z}_a^{\tau}$. Moreover, the previous argument shows that $\widehat{z}_a^{TIF}$ lies below the socially-optimal level z_a^*. Therefore,

$$\text{incremental TIF tax revenue} \quad = \quad \frac{\tau+t}{\tau+t+r}R_a' \quad > \quad C_a' \quad \text{when} \quad z_a \quad < \quad \widehat{z}_a^{TIF}, \tag{19}$$

where[9]

$$\widehat{z}_a^{\tau} \quad < \quad \widehat{z}_a^{TIF} \quad < \quad z_a^*. \tag{20}$$

The inequality in (19) reversed, indicating that TIF is not viable, when $z_a > \widehat{z}_a^{TIF}$.

This discussion shows that TIF is viable, with incremental TIF tax revenue exceeding the cost of the public improvement, when $z_a < \widehat{z}_a^{TIF} < z_a^*$. Moreover, TIF is needed, in the sense that city revenue alone cannot cover the cost of the improvement, when $z_a > \widehat{z}_a^{\tau}$. Thus, TIF's "range of relevance," where it is both needed and viable, consists of initial z_a values lying in the interval $[\widehat{z}_a^{\tau}, \widehat{z}_a^{TIF}]$, whose endpoints are $\widehat{z}_a^{\tau}$ and the TIF-critical value. The range of relevance is shown in Figure 1, which also illustrates the taxonomy of other possible cases. Note that when the initial z_a lies below $\widehat{z}_a^{\tau}$, then the first inequality in (16) is reversed, and the city tax revenue generated by the public improvement exceeds its cost, allowing a reduction in τ. Although TIF is viable in this case given $z_a < \widehat{z}_a^{\tau} < \widehat{z}_a^{TIF}$, it is unneeded since the public improvement generates no opposition. The next interval in Figure 1 is the range of relevance $[\widehat{z}_a^{\tau}, \widehat{z}_a^{TIF}]$, where city tax revenue does not cover the cost of the improvement, but where TIF is viable. In this situation, where the public good is moderately underprovided, use of TIF facilitates

the public improvement, which otherwise would not take place. When z_a is instead slightly underprovided, with z_a lying between $\widehat{z}_a^{TIF}$ and z_a^*, TIF is needed to finance the improvement, but it is not viable. The same conclusion applies when the public good is initially overprovided, with $z_a > z_a^*$.

Figure 1 suggests an important conclusion, namely that TIF cannot be used to finance a marginal public improvement when one is not socially desirable. This conclusion, which follows because TIF is not viable when z_a is initially above z_a^*, suggests that TIF is not a means for carrying out misguided public projects. As seen below, this conclusion must be qualified when *nonmarginal* public improvements are considered. The converse of the previous statement, however, does not hold: TIF fails in some situations where a public improvement would be desirable. This outcome occurs when the public good is slightly underprovided, with the initial z_a lying between $\widehat{z}_a^{TIF}$ and z_a^*, in which case an improvement is warranted but TIF is not viable. The conclusion to be drawn is that TIF can facilitate a public improvement in some, but not all, cases where one is socially desirable.

Summarizing yields

Proposition 2. *When the public good in a neighborhood is moderately underprovided, TIF can be used to finance a marginal public improvement, obviating the need for an unpopular tax increase. However, TIF is not a viable financing method when the public good is only slightly underprovided. Thus, TIF is viable in some, but not all, cases where a public improvement is socially desirable.*

It is important to note that Proposition 2 may be overturned when condition Γ from above is not satisfied. When this condition does not hold, a positive critical value $\widehat{z}_a^{\tau}$ does not exist, and it is easily seen that the same conclusion may also apply to $\widehat{z}_a^{TIF}$. With both critical values nonexistent, TIF's range of relevance is then empty, implying that TIF is *never viable* as a means of financing the public improvement. To avoid this outcome, ensuring a nonempty range of relevance, it must be true that the social benefit from the first unit of z_a is large relative to its cost, as discussed above.

3.2. Comparative-static analysis of TIF's range of relevance

It is interesting to explore the dependence of TIF's range of relevance on the underlying

parameters, focusing in particular on the effect of the school tax rate t. Consider the case where the school district provides the same public good level s but charges a higher tax rate t. Such a difference might result from a lower level of state support for the district, which means that t must be higher to support the same level of expenditure.

To find the effects of the higher t, consider the previous equations defining the endpoints $\widehat{z}_a^{\tau}$ and $\widehat{z}_a^{TIF}$ of the range of relevance: $\tau R_a'(\widehat{z}_a^{\tau})/(\tau+t+r) - C_a'(\widehat{z}_a^{\tau}) = 0$ and $(\tau+t)R_a'(\widehat{z}_a^{TIF})/(\tau+t+r) - C_a'(\widehat{z}_a^{TIF}) = 0$. Recall from above that the left-hand side expressions in both of these equations are locally decreasing in the level of the public good. As a result, comparative-static analysis shows that when t rises, the change in $\widehat{z}_a^{\tau}$ is in the same negative direction as the change in the multiplicative factor $\tau/(\tau+t+r)$. Similarly, when t rises, the change in $\widehat{z}_a^{TIF}$ is in the same positive direction as the change in $(\tau+t)/(\tau+t+r)$. Therefore,

$$\frac{\partial \widehat{z}_a^{\tau}}{\partial t} < 0 \; ; \qquad \frac{\partial \widehat{z}_a^{TIF}}{\partial t} > 0, \tag{21}$$

indicating that TIF's range of relevance widens as the school tax rate increases.[10] The intuitive explanation is that an increase in t reduces property values for a given τ, thus depressing the city's own tax-revenue gain from the public improvement. With a smaller revenue gain at any initial value of z_a, TIF is needed sooner (at lower values of z_a) when t is higher. Conversely, when t is higher, the public improvement leads to a larger increase in *combined* city and school revenue under TIF. As a result, the higher t makes TIF viable up to a larger value of z_a.

The results in (21) can be generalized by imagining that instead of being contained in one overlapping school district, the TIF zone is instead contained in n overlapping jurisdictions, whose average property tax rate equals $\overline{t}$. In this case, all the previous analysis applies, provided that t is replaced by $n\overline{t}$. Since the individual effects of n and $\overline{t}$ are the same as those of t above, the following result applies:

Proposition 3. *TIF's range of relevance widens as the school tax rate t increases. In the case where the TIF zone is contained in many overlapping jurisdictions, the range of relevance widens as their average tax rate increases, or as the number of jurisdictions n rises.*

Conforming to intuition, the Proposition shows that TIF is viable up to a higher level of z_a when number of jurisdictions providing captured tax revenue is larger. However, by generating a larger tax burden and thus lower values, a larger n reduces the city's own incremental tax revenue from an improvement, reducing $\widehat{z}_a^{\tau}$ and heightening the need for TIF.

3.3. Analysis of nonmarginal public improvements

Up to this point, the analysis has focused on marginal public improvements, which involve only a small increase in z_a. This is appropriate given that a principal goal has been to evaluate the viability of TIF, which requires comparing the tax-revenue increase from a marginal improvement to its cost. Once viability is established, however, an additional question arises: how large should the public improvement be?

Because (18) holds when the initial z_a lies in TIF's range of relevance, it follows that a marginal improvement generates a *budget surplus* for the city. Given the presence of this excess tax revenue, the post-improvement situation cannot represent the ultimate equilibrium. To see what the equilibrium must involve, note that because the property-tax rates τ and t remain constant under TIF, property values in the TIF zone rise as z_a increases, a consequence of higher rents in the zone (values in neighborhood b are unaffected). Therefore, pursuing the interests of property owners, the city should increase z_a *as far as possible*, being constrained only by the balanced-budget requirement. This means that z_a should keep increasing until the budget surplus that emerges following the initial marginal improvement is gone. Thus, while a z_a value in the range of relevance represents a *starting point* for a feasible implementation of TIF, the z_a level ultimately chosen under TIF is larger by a nonmarginal amount. As in the case of a marginal improvement, political opposition to the nonmarginal improvement is defused by keeping τ constant as z_a increases.

To formalize this discussion, let λ represent the increase in z_a above its initial level. Then, consider the following expression, which gives the city's incremental tax revenue from the improvement minus incremental costs, as a function of λ:

$$M(\lambda) \equiv \frac{\tau + t}{\tau + t + r}[R_a(z_a + \lambda) - R_a(z_a)] - [C_a(z_a + \lambda) - C_a(z_a)]. \tag{22}$$

Note that revenues and costs in neighborhood b are constant and can be ignored in deriving (22). Inspection of (22) shows that $M(0) = 0$. Moreover, since the tax rates remain constant, differentiation of (22) yields

$$M'(\lambda) \;=\; \frac{\tau + t}{\tau + t + r} R_a'(z_a + \lambda) \;-\; C_a'(z_a + \lambda), \tag{23}$$

which can take either sign. Note, however, that $M'(0) > 0$ holds when z_a lies in TIF's range of relevance, as assumed ((18) then holds). Finally differentiation of (23) yields $M'' = (\tau + t)R_a''/(\tau + t + r) - C_a'' < 0$. These conclusions establish that $M(\cdot)$ is a strictly concave function that passes through the origin, where it is increasing, as shown in Figure 2. Note that if z_a were less than $\widehat{z}_a^{TIF}$ instead of lying in the range of relevance, then the M curve would instead be downward sloping at the origin, indicating that any increase in z_a generates a budget deficit. This case is shown by the dotted curve in Figure 2.

Since the city's budget was balanced prior to the public improvement, it follows that, as λ increases away from zero, a budget surplus emerges and initially grows as M rises above the horizontal axis in Figure 2. The surplus achieves a maximum at the peak of the M curve, where (23) equals zero. Recalling (19), it follows that the maximum is achieved at a z_a value equal to $\widehat{z}_a^{TIF}$. The surplus then falls, reaching zero when $\lambda = \lambda^{**}$. Because property values are increasing in z_a, λ^{**} and the associated public-good level $z_a^{**} = z_a + \lambda^{**}$ are therefore optimal from the city's point of view. They lead to the highest possible property values in the TIF zone consistent with maintenance of a balanced budget.

An important question concerns the relationship between z_a^{**} and the socially-optimal level z_a^*. To investigate this relationship, observe that λ^{**} (and hence z_a^{**}) in Figure 2 must lie beyond the maximum of the M curve. But since $z_a = \widehat{z}_a^{TIF}$ holds at the curve's maximum, it follows from (20) that z_a^* also lies above the maximum. As a result, both z_a^* and z_a^{**} lie above the maximum of the M curve, and this conclusion means that their magnitudes cannot be compared. Therefore, z_a^{**} could lie below or above the socially-optimal level z_a^*.[11] Summarizing yields

Proposition 4. *If the initial public-good level lies in TIF's range of relevance, then a marginal public improvement generates a budget surplus. When the city's goal is property-value maximization, it should then undertake a nonmarginal increase in z_a up to the point where the budget surplus is eliminated. The resulting public-good level could lie above or below the socially-optimal level.*

The preceding discussion shows that, while TIF provides a viable method for increasing the level of an initially-underprovided public good, the ultimate outcome may be overprovision. Thus, TIF may allow a city to carry out needed public improvements, but the stimulus it provides may be excessive. In other words, while a *marginal* improvement is socially desirable when z_a initially lies in the range of relevance, as seen in Figure 1, the *nonmarginal* improvement ultimately chosen under TIF may be too large.

The solution, of course, is to abandon TIF and instead choose public-good levels and tax rates to maximize aggregate property value in the city. While this approach guarantees an efficient outcome, as seen above, it may encounter political resistance from property owners who suffer losses amidst the generalized gains. TIF represents an imperfect response to this opposition.

A final result concerns the effect of the school district's tax rate on z_a^{**}. Since λ^{**} satisfies $M(\lambda^{**}) = 0$, differentiation shows that $\partial\lambda^{**}/\partial t = -\Delta/M'(\lambda^{**})$, where Δ denotes the positive derivative of M with respect to t. Since $M'(\lambda^{**}) < 0$ must hold from Figure 2, it follows that

$$\frac{\partial\lambda^{**}}{\partial t} \quad > \quad 0. \tag{24}$$

Thus, by providing more revenue, a larger school tax rate allows z_a to be increased to a higher level before the city encounters a budget deficit. Generalizing this result to the case of many overlapping jurisdictions yields

Proposition 5. *The public-good level z_a^{**} chosen under TIF rises as the school tax rate t increases. In the case where the TIF zone is contained in many overlapping jurisdictions, z_a^{**} rises as their average tax rate increases, or as the number of jurisdictions n increases.*

Thus, as intuition would suggest, a TIF zone that captures revenue from many overlapping jurisdictions ends up with a relatively high public-good level.

3.4. Generalization to the case of endogenous housing attributes

Up to this point in the analysis, housing attributes have been held fixed, precluding an investment response to the public improvement. Given that a principal expectation underlying TIF is that the public improvement it finances will generate new construction and upgrading of existing property, this restriction is undesirable. However, as explained above, generalizing the model to allow an investment impact has no effect on the results.

To see this, let the rent function for house i be rewritten as $\widetilde{P}_i(z_a, q_i)$ to include the now-endogenous housing attributes q_i, which for simplicity are represented by a scalar rather than a vector. Letting c denote the annualized unit cost of q, the net rent from house i is then given by $\widetilde{P}_i(z_a, q_i) - cq_i$. Conditional on z_a, the owner chooses q_i to maximize net income, which requires $\partial\widetilde{P}_i/\partial q_i = c$, or $U_i^q/U_i^x = c$. The resulting optimal attribute level is a function of z_a, being written $q_i(z_a)$, and its derivative is likely to be positive (its sign depends on the properties of preferences). Substituting $q_i(z_a)$, net income from house i can then be written $\widetilde{P}_i(z_a, q_i(z_a)) - cq_i(z_a) \equiv P_i(z_a)$.

The function $P_i(\cdot)$ now gives the effect of z_a on net rent from the house, taking into account the public good's indirect effect on housing attributes. Using the envelope theorem, its derivative is given by $P_i'(z_a) = \partial\widetilde{P}_i/\partial z_a = U_i^z/U_i^x = \mathrm{MRS}_i^{z,x}$. Therefore, the effect of the public good on (net) rent continues to be given by the previous $\mathrm{MRS}_i^{z,x}$ expression, implying that the total rent derivative R_a' in (2) is unchanged. Moreover, it can be shown that $P_i(z_a)$ continues to be a concave function of z_a, preserving concavity of the total rent functions. The upshot is that the preceding analysis is entirely unchanged when investment effects are added. These effects operate behind the scenes, with their impacts subsumed in the previous formulas via the envelope theorem. The analysis can also be generalized to include nonresidential property, following the approach of Brueckner and Wingler (1984).[12]

4. Conclusion

This paper offers an analysis of tax increment financing, adding to a small theoretical literature on this important fiscal instrument. The analysis exploits the theoretical connection between property values and public-good levels, which is the subject of a large literature in

local public finance. Using this approach, the paper shows that localized public improvements are likely to be opposed by property owners outside the affected area, who pay higher property taxes with no offsetting benefits. By using tax revenue captured from overlapping jurisdictions, TIF may circumvent this opposition, allowing the city to implement the public improvement without an increase in its tax rate. TIF is not always viable as a financing method, however, because it may not generate enough additional revenue. The analysis shows that TIF's viability is ensured only when the public good is at least moderately underprovided relative to the socially-optimal level. In the case where the public good is slightly underprovided, a public improvement is desirable, but TIF is not viable. Finally, the analysis shows that public-good levels ultimately chosen under TIF need not be efficient, with both under- and overprovision being possible outcomes. Thus, while TIF may allow a city to carry out needed public improvements, the stimulus it provides may be excessive.

The present model could be altered in a number of ways. For example, public-good spillovers across neighborhoods could be introduced. An effect of this modification would be to dilute the political opposition arising from neighborhood-b landowners, whose tax increase would now be accompanied by spillover benefits from the improvement. As a result, the need for TIF would be reduced.

In addition, the model could be developed in an intertemporal context to capture secular growth in property values. A natural way of doing so would be to assume exogenous exponential growth in rents, with total rent in neighborhood a at time T given by $R_a(z_a)e^{gT}$, and similarly for neighborhood b. In this case, total property value in the city at time T is given by $[R_a(z_a)+R_b(z_b)]e^{gT}/(\tau+t+r-g)$. Note that r now represents the nominal discount rate, with $r-g$ giving the "real" rate. Assuming that public-good costs also grow at rate g, budget balance for the city at T requires that τ times the above expression equals the cost term on right-hand side of (5) times e^{gT}. Since the exponential terms in this new equation cancel, the budget-balancing τ is constant over time but now depends on the real discount rate. The basic model is thus unchanged in an intertemporal context, and the remaining analysis unfolds as before provided that a certain assumption is imposed. In particular, it must be assumed that TIF allows the city to capture *only* the increase in tax revenue caused by the improvement, with the

school district retaining the secular part of the revenue increase. In other words, in a TIF zone established at time T_0, the city's incremental tax revenue at time T is $(\tau+t)R_a' e^{gT}/(\tau+t+r-g)$. If the city were also to capture the secular revenue growth, contrary to assumption, then the increment would be larger by $(\tau+t)R_a(e^{gT}-e^{gT_0})/(\tau+t+r-g)$.

Given that all incremental tax revenue from the TIF zone, including that due to secular increases, is captured by the city in actuality, the above scenario is unrealistic. Thus, in contrast to the above assumption, the revenue streams of overlapping jurisdictions are completely frozen under TIF, while operating costs may be rising over time. Budget pressure may then arise, creating opposition to TIF among overlapping jurisdictions. This opposition is partly lessened, however, by the fact that TIF authority usually expires after a fixed period, at which point these jurisdictions enjoy a revenue windfall.

The above issues could be explored further in a fully-specified intertemporal model. Given the importance of TIF as a local policy tool, additional theoretical and empirical research exploring its effects deserves high priority.

Marginal improvement is socially desirable	YES	YES	YES	NO
Effects of improvement without TIF on:				
Neighborhood a owners	GAIN	GAIN	GAIN	?
Neighborhood b owners	GAIN	LOSE	LOSE	LOSE
TIF needed to implement improvement	NO	YES	YES	YES
TIF viable	YES	YES	NO	NO

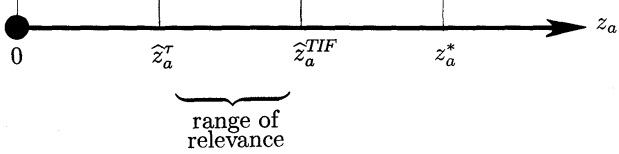

Figure 1: Taxonomy of Cases

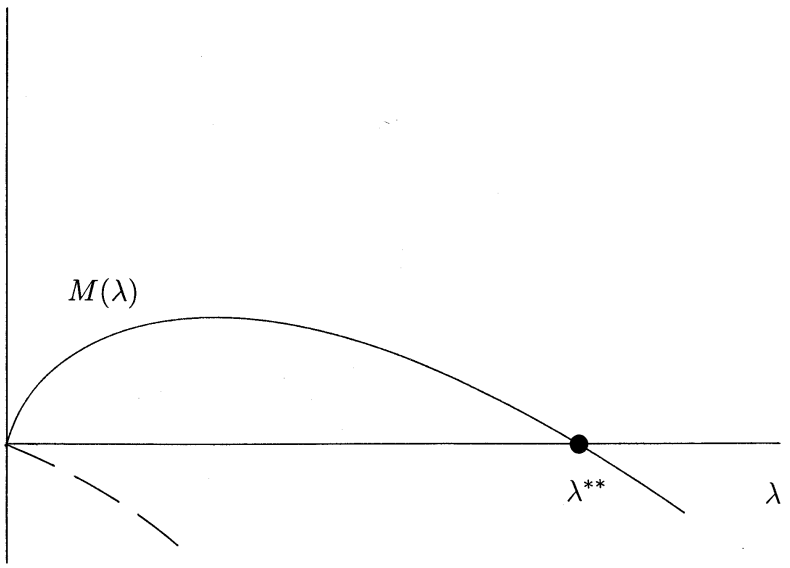

Figure 2: The M function

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Footnotes

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[1]In the calculation, (5) is used to eliminate τ. While the approach used in (7) is appropriate given the emphasis of the present analysis, a simpler way of deriving the efficiency results is to eliminate the tax rates from the expression for aggregate property value. Eliminating $(\tau + t)(V_a + V_b)$ from the equation $V_a + V_b = [R_a + R_b - (\tau + t)(V_a + V_b)]/r$ using the combined budget constraints of the city and school district, the equation reduces to $V_a + V_b = [R_a(z_a) + R_b(z_b) - C_a(z_a) - C_b(z_b) - K(s)]/r$. Maximizing this expression with respect to the public good levels directly yields the relevant Samuelson conditions.

[2]After using (5) to substitute $(C_a + C_b)/(R_a + R_b)$ in place of $\tau/(\tau+\theta)$ in $\tau R_a'/(\tau+\theta) - C_a' \equiv \Phi$, the derivative of Φ with respect to z_a has the sign of

$$[R_a''(C_a + C_b) - C_a''(R_a + R_b](R_a + R_b) \;-\; R_a'(R_a + R_b)\left[\frac{\tau}{\tau+\theta}R_a' - C_a'\right].$$

The first term is negative, and the second is negative or zero when $\Phi \geq 0$. Therefore, since $d\Phi/dz_a < 0$ holds when $\Phi \geq 0$, it follows that Φ changes sign at most once, from positive to negative.

[3]Using the previous expression for τ and assuming $C_a(0) = 0$, $\tau_0 = \theta C_b(z_b)/[R_a(0) + R_b(z_b) - C_b(z_b)]$. Note that while $R_a(0) < 0$ could hold in principle, indicating that a negative total rent is required for residents of neighborhood a to achieve the given utility level in the absence of the public good, this outcome is ruled out as unrealistic.

[4]Total differentiation of $\alpha R_a'(z_a) - C_a'(z_a) = 0$ yields $\partial z_a/\partial\alpha = -R_a'/(\alpha R_a'' - C_a'') > 0$, which establishes (12).

[5]Note that if condition Γ is not satisfied, then Proposition 1 is altered. The derivative $\partial\tau/\partial z_a$ is then positive regardless of the initial level of z_a, so neighborhood b property owners are always harmed by the improvement.

[6]This increase in aggregate value emerges because the increase in neighborhood a is sufficient to offset the decline in b.

[7]Since τ is raised along with z_a in the previous case, the increase in aggregate value (and

hence the size of the incremental school-district surplus) is smaller than in the present case, where τ is held constant as z_a rises.

[8]Using (5) and the school district's budget constraint, $(\tau+t)/(\tau+t+r)$ equals $(C_a+C_b+K)/(R_a+R_b)$. Substituting this expression into $(\tau+t)R_a'/(\tau+t+r)-C_a' \equiv \Lambda$, the derivative of Λ with respect to z_a has the sign of

$$[R_a''(C_a+C_b+K)-C_a''(R_a+R_b](R_a+R_b) \;-\; R_a'(R_a+R_b)\left[\frac{\tau+t}{\tau+t+r}R_a'-C_a'\right].$$

The first term is negative, and the second is negative or zero when $\Lambda \geq 0$. Therefore, since $d\Lambda/dz_a < 0$ holds when $\Lambda \geq 0$, it follows that Λ changes sign once, from positive to negative ($\Lambda > 0$ holds at $z_a = 0$ when condition Λ is satisfied).

[9]To establish (20), observe that $\widehat{z}_a^{TIF}$ satisfies the equation $\beta R_a'(z_a) - C_a'(z_a) = 0$, where $\beta = (\tau+t)/(\tau+t+r) < 1$, while z_a^* satisfies the same equation with β replaced by 1. By applying the argument used above to locate $\widehat{z}_a^{\tau}$, it follows that $\widehat{z}_a^{TIF} < z_a^*$. To compare $\widehat{z}_a^{TIF}$ to $\widehat{z}_a^{\tau}$, recall that $\widehat{z}_a^{\tau}$ satisfies the equation $\alpha R_a'(z_a) - C_a'(z_a) = 0$, where $\alpha = \tau/(\tau+t+r)$. Since $\beta > \alpha$, the previous argument then establishes $\widehat{z}_a^{TIF} > \widehat{z}_a^{\tau}$. It should be noted that the value of τ in the equalities defining β and α is the same for a given z_a, a fact that is required to conclude that $\beta > \alpha$. This is true because τ is initially set to balance the city's budget, regardless of whether or not TIF is used to finance an increase in z_a.

[10]Holding t fixed, a higher s also affects the range of relevance through its impact on τ. Because a higher s raises rents in both neighborhoods, the τ required to support a given initial z_a falls. It is easily seen that this reduction in τ shifts the range of relevance to the left.

[11]Given that z_a^* is characterized by a condition involving the derivative of R_a while z_a^{**} is characterized by the condition $M(\lambda^{**}) = 0$, which involves the level of R_a, it appears impossible to derive a simple comparison of their magnitudes.

[12]To see this, let the strictly-concave function $F_j(z_a, b_j, q_j)$ denote the output of factory j in neighborhood a, which depends on the public-good level, the real estate input q_j, and a variable input b_j, whose unit cost is normalized to one. Assuming that q_j is fixed and letting p denote the output price, the factory chooses b_j to maximize net income $pF_j(z_a, b_j, q_j) - b_j$, satisfying $p\partial F_j/\partial b_j = 1$. Substituting the optimal value $b_j(z_a)$, net income is then $W_j(z_a) \equiv pF_j(z_a, b_j(z_a), q_j) - b_j(z_a)$, a strictly concave function. Using the envelope theorem and eliminating p via the above first-order condition, it follows that $W_j'(z_a) = (\partial F_j/\partial z_a)/(\partial F_j/\partial b_j) \equiv \mathrm{RTS}_j^{z,b}$, where $\mathrm{RTS}_j^{z,b}$ gives the rate of technical substitution between the public good and the variable input for factory j. When neighborhood a has a mixture of residential and nonresidential property, total rent is given by $R_a(z_a) = \sum_{i\in a} P_i(z_a) + \sum_{j\in a} W_j(z_a)$, and its derivative is $R_a'(z_a) = \sum_{i\in a} \mathrm{MRS}_i^{z,x} + \sum_{j\in a} \mathrm{RTS}_j^{z,b}$. The

socially-optimal level of z_a is determined by setting this expression equal to $C_a'(z_a)$, which yields a "generalized" Samuelson condition (see Brueckner and Wingler (1984) for further discussion). With these alterations, the remainder of the previous analysis is unchanged, which shows that the model easily embraces the addition of nonresidential property. Note that the nonresidential real estate input could also be made endogenous, as above.